

The Use of Technology in the Teaching and Learning of Mathematics Framework

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The Didactic Triangle

The didactic triangle is a representation that has been used by several researchers (e.g., Brousseau, 1997; Freudenthal, 1991; Steinbring, 2005) to describe interactions that occur among a teacher, his or her students, and the content that is being taught. These interactions can be described in terms of pedagogical activities the teacher uses to engage students in learning content - in this case, mathematics. It is important to note that when we refer to mathematics we are considering mathematical topics like algebra, geometry, measurement, statistics, probability, number, and operations and also the mathematical processes students use when engaging with mathematics. These include using representations, making connections, communicating reasoning, creating and critiquing arguments, attending to precision, solving problems, and mathematical modeling (NCTM, 2000; 2014; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010)

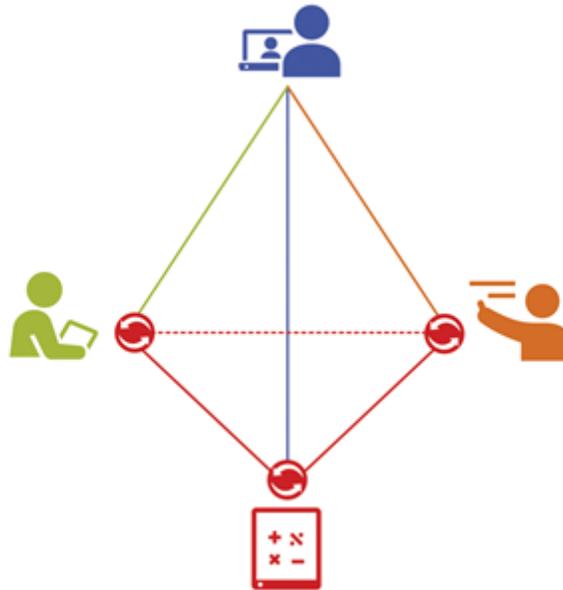
We will focus on interactions that are planned and used by the teacher. These include (but are not limited to) pedagogical activities related to 1) the selection and implementation of mathematical tasks, 2) questions teachers pose to push student thinking or probe their understandings, 3) strategies teachers use to facilitate mathematical discussions, and 4) methods teachers use to assess what students are thinking and learning. We depict these four pedagogical activities at the center of our didactic triangle.



While there are many factors that influence classroom interactions such as classroom norms, classroom culture, student attitudes, and beliefs, these will not be the focus of this particular course.

The Didactic Tetrahedron

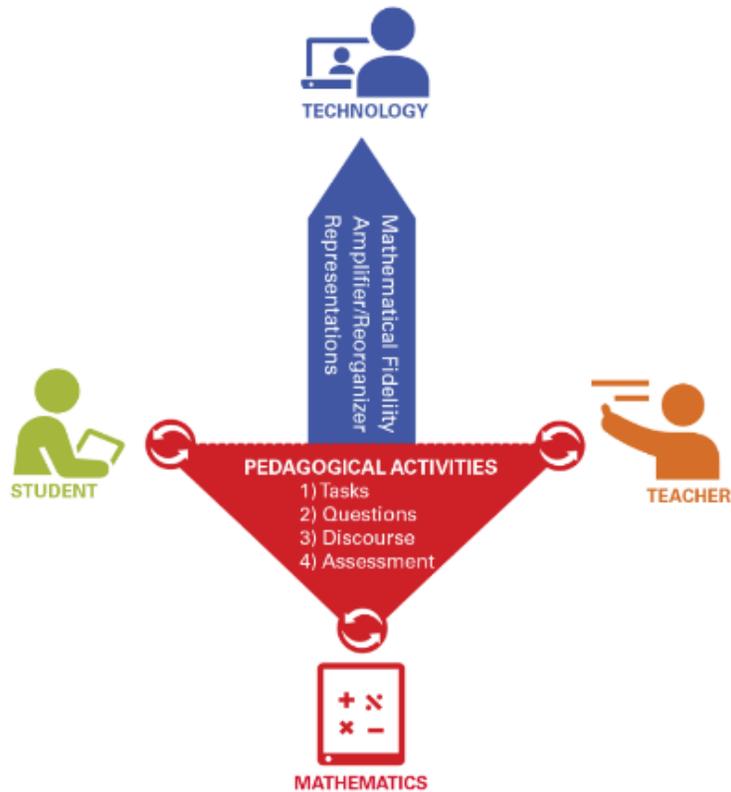
When technology is used in a mathematics classroom, it influences all four of those pedagogical activities. To make explicit how one considers the role of technology among interactions with students, a teacher, and mathematics, the didactic triangle was extended by Tall (1986), and more recently by Olive et al. (2010) and Ruthven (2012). We can depict this influence by expanding our didactic triangle to create a didactic tetrahedron with technology as the fourth vertex.



Technology offers unique affordances for teachers and students to use when teaching and learning mathematics. Some technological affordances include the number crunching power of a spreadsheet to assist students in discovering the value of e . Or the ability to quickly generate and change different representations of functions to support students in understanding relationships between the symbolic and graphical representations of functions.

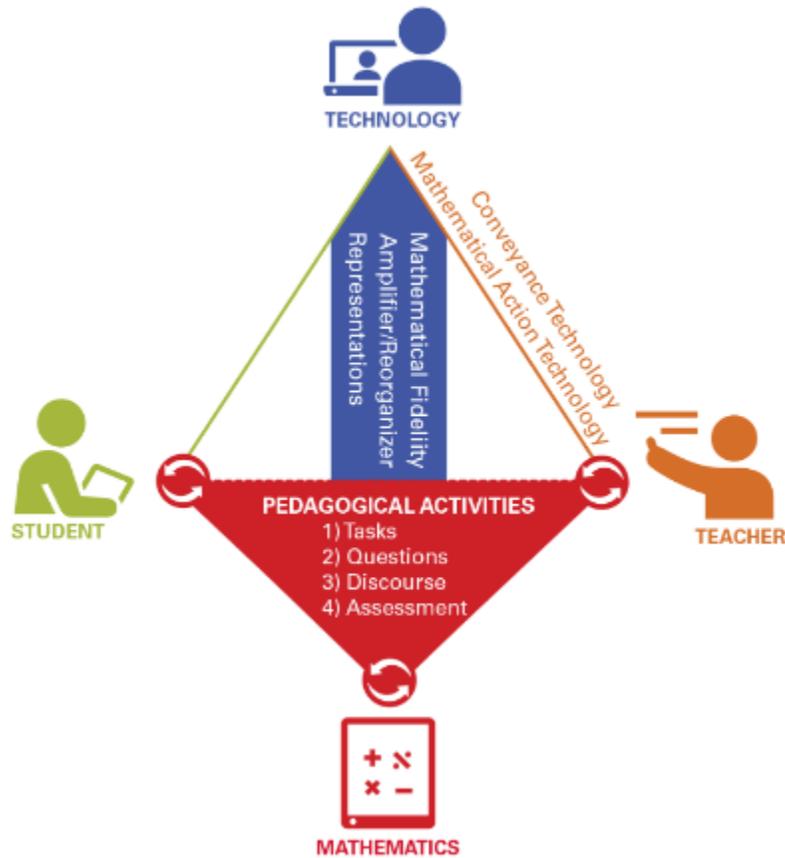
Interactions among the Teacher, Student, Technology, and Mathematics

We can describe these affordances by focusing on the edges of the tetrahedron that depict interactions between technology and mathematics, technology and the teacher and technology and the student. We pose questions a teacher might ask him or herself when making a decision about whether to use a particular technology tool in the mathematics classroom.

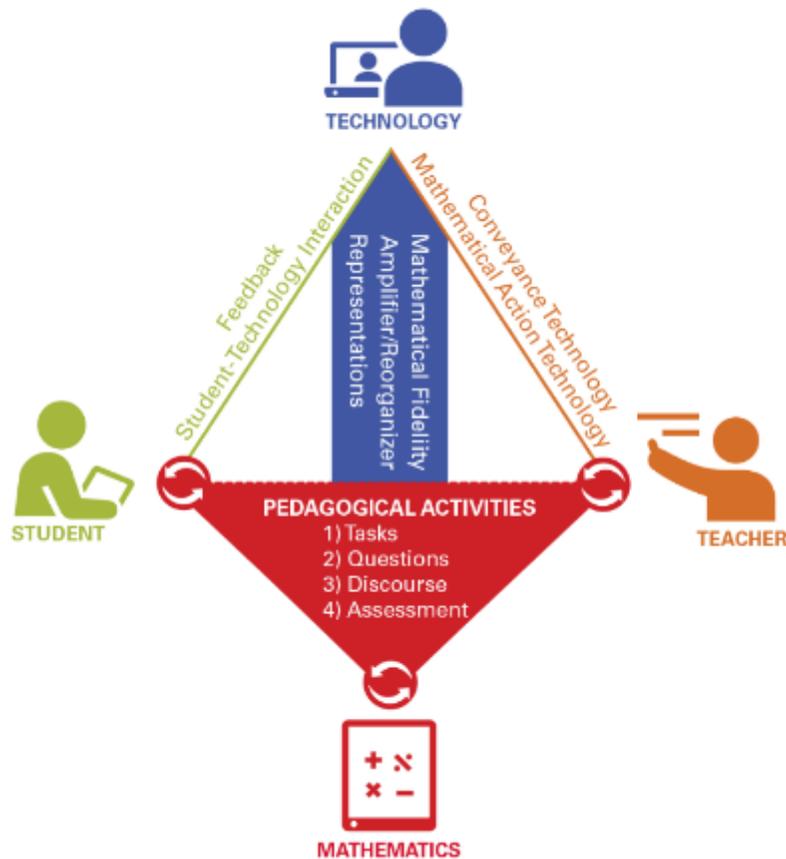


<p><i>Technology-Mathematics</i></p>	<ol style="list-style-type: none"> 1) Mathematical Fidelity. Is the technology tool a faithful and true representation of the mathematics students are to learn? (Dick, 2008) 2) Amplifier/Reorganizer. Does the technology allow the teacher and/or student to do the same work more effectively, efficiently, and quickly (amplifier)? Does the technology change the way the student and/or teacher thinks about mathematical ideas (reorganizer)? (Pea, 1985) 3) Representations. How does the technology represent the mathematics? Does it provide linked representations for students to use? (Goldin & Kaput, 1995)
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Technology-Teacher Interactions



Technology Consideration	Issues to Consider and Questions to Pose
Technology-Teacher	<ol style="list-style-type: none"> 1) Conveyance/Mathematical Action Technology. Will the technology be used for the teacher to convey information to students (e.g., power point, internet)? Will the technology be used to allow students to perform mathematical actions? (Dick & Hollebrands, 2011) 2) Is the technology readily available for the teacher? Is the learning curve minimal for the teacher?



Technology-Student	<ol style="list-style-type: none"> 1) Interaction. How does the student interact with the technology? Is the technology available? Is the technology learning curve minimal or steep? 2) Feedback. What types of feedback does the technology provide to students when they are interacting with it?
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It is important for the teacher to be aware of the opportunities technology allows and consider how they influence the four pedagogical activities involved in 1) designing tasks, 2) posing questions, 3) facilitating discourse, and 4) assessing student learning. We present these as questions a teacher may consider when using technology in the mathematics classroom.

Pedagogical Activities	Questions to Consider
Designing Tasks	<ol style="list-style-type: none"> 1) What is the cognitive demand of this technology-based task? (Stein & Smith, 1998) 2) How will the student interact with the task and technology? 3) How does technology enhance student learning? 4) What learning goals would be best served by this task? 5) How might I prepare students to engage productively in this task?
Questions	<ol style="list-style-type: none"> 1) What new questions does this technology allow me to ask? 2) In what ways can I ask questions that will advance student thinking and probe what students are learning? 3) What opportunities does the technology allow for students to pose their own mathematical questions? 4) How might I structure my classes to help students feel comfortable generating and posing their own questions and responding to questions that other students generate?
Discourse	<ol style="list-style-type: none"> 1) Does the technology allow for different solutions and/or different solution strategies? 2) What would make a discussion of technology-based tasks productive? 3) How can I use technology to facilitate a productive mathematics discussion?
Assessment	<ol style="list-style-type: none"> 1) What type of feedback does the technology provide to the student? 2) How can I build self-assessment into the tasks? 3) How can I leverage the technology to determine what students are learning? 4) How can I use the technology to assess what the students have learned?

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